Written Exam at the Department of Economics winter 2020-21

Contract Theory

Final Exam

22 January 2021

(4-hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 5 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

Attempt both questions.

Explain all the steps of your analysis and define any new notation that you use.

Show all the calculations that your analysis relies on.

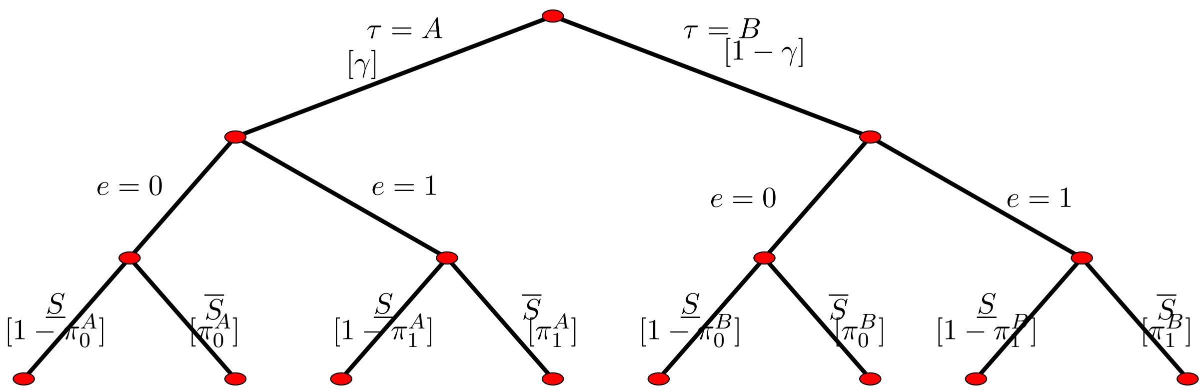
Question 1: Private information about both the choice and importance of effort

The following model is an extension of the 2x2 moral hazard model with a risk neutral agent (protected by limited liability) that we studied in the course. In the extension, the agent has private information about the nature of the project that she is employed to work on.

Consider a moral hazard model where the principal and the agent are both risk neutral but where the agent is protected by limited liability. The project gives rise to either a large surplus ($S = \overline{S}$) or a small surplus ($S = \underline{S}$, with $\overline{S} > \underline{S} > 0$). The likelihood of each outcome depends on whether the agent chooses a low effort ($e = 0$) at no cost, or a high effort ($e = 1$) at cost $\psi > 0$. However, how much of a difference it makes to choose the high effort level depends on the project's type, $\tau \in \{A, B\}$. Specifically, given an effort level $e \in \{0, 1\}$ and a project type $\tau \in \{A, B\}$, the probability that a large surplus is realized equals $\pi_e^\tau \in (0, 1)$. (Accordingly, the probability of a small surplus is $1 - \pi_e^\tau$.) The following relationships hold:

$$\pi_1^A - \pi_0^A > \pi_1^B - \pi_0^B > 0.$$

That is, for both project types, choosing a high instead of a low effort increases the likelihood of a large surplus, although doing this is most helpful with an A project. The agent can, before choosing her effort level, observe the project's type (A or B) perfectly. The principal, however, cannot observe the project type and assigns the probability $\gamma \in (0, 1)$ to the event that $\tau = A$. The figure below illustrates some of the assumptions made in the text.



The principal can commit to making a monetary payment t to the agent that is contingent on the observed outcome. In particular, the assumptions made above mean that the payment can be contingent on two different events:

- $S = \underline{S}$ (denote the payment after this event by $\underline{t}$); and
- $S = \overline{S}$ (denote this payment by $\overline{t}$).

As the agent is protected by limited liability, the following two conditions must hold:

$$\underline{t} \geq 0 \quad \text{and} \quad \overline{t} \geq 0. \tag{LL}$$

The timing of events is as follows. (i) The principal chooses $(\underline{t}, \overline{t})$, trying to maximize the expected surplus that is generated, net of the expected payments to the agent. (ii) The agent, who does not yet know the type τ of the project, decides whether or not to accept the contract offer. Her objective is to maximize her expected payment t, minus the effort cost (if any). (iii) If not accepting, the agent receives her outside option payoff, which is zero (and the principal receives the same zero payoff). If the agent accepts the offer, she first observes the type τ of the project and then chooses the effort level $e_\tau \in \{0, 1\}$. If the agent accepted the offer at stage (ii), she is committed to this and is not, after having learned τ, allowed to leave the interaction with the principal.

Answer the following questions.

(a) Suppose the principal wants to induce the outcome $e_A = 1$ and $e_B = 0$. What are the optimal choices of $\underline{t}$ and $\overline{t}$? You are encouraged to show your results by using a graphical analysis.

(b) Suppose there are no limited liability constraints in this model (meaning that any $\underline{t} \in \mathbb{R}$ and $\overline{t} \in \mathbb{R}$ are allowed). With this assumption, can the principal induce the outcome $e_A = 1$ and $e_B = 0$ without giving away rents to the agent? Explain why or why not. Also explain any differences in the reasoning and the conclusions relative to the model we studied in the course (i.e., the 2x2 moral hazard model with a risk neutral agent who is not protected by limited liability, and with *known* probabilities of a large surplus, with and without effort).

Question 2: Adverse selection in a competitive insurance market

The following is a model of a competitive insurance market with adverse selection. It builds on similar models that we studied in the course.

A car owner (the agent in our model, A) considers to purchase a car insurance, as this would give her financial protection in case she had an accident. The likelihood of an accident depends on A's type.

A skillful (and therefore a *low*-demand) driver has an accident with probability $\underline{\theta}$, and a less skillful (and therefore a *high*-demand) driver has an accident with probability $\overline{\theta}$, where $0 < \underline{\theta} < \overline{\theta} < 1$. The market is characterized by asymmetric information, which here means that the insurance company that A considers to purchase her insurance from (P) does not know A's type, but assigns the probability $\nu \in (0, 1)$ to the event that $\theta = \underline{\theta}$. However, A knows her own type perfectly.

A's disutility of having an accident, measured in monetary terms as a deduction from her income, is denoted by $d > 0$, and A's monetary income is denoted by w (with $w > d$). Moreover, A's payment to the insurance company (P) in case there is no accident is denoted by p; and the *net* compensation A receives from P in case there indeed is an accident is denoted by a. A is risk averse and her utility function is denoted by u (where $u' > 0$ and $u'' < 0$). Therefore, A's utility if purchasing the insurance is

$$\begin{cases} u(w - d + a) & \text{if having an accident} \\ u(w - p) & \text{if not having an accident.} \end{cases}$$

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or "lower-bars", depending on which type the contract is aimed at. The contract variables are p and a. However, to solve the problem it is more convenient to think of P as choosing the utility levels directly, instead of the contract variables. We thus introduce the following notation:

$$\overline{u}_N \overset{\text{def}}{=} u(w - \overline{p}), \quad \overline{u}_A \overset{\text{def}}{=} u(w - d + \overline{a}), \quad \underline{u}_N \overset{\text{def}}{=} u\left(w - \underline{p}\right), \quad \underline{u}_A \overset{\text{def}}{=} u(w - d + \underline{a}).$$

Also, let h be the inverse of u (hence $h' > 0$ and $h'' > 0$). In terms of the new notation, P's expected profit can be written as

$$\begin{aligned} V &= \nu\left[(1 - \underline{\theta})\underline{p} - \underline{\theta}\underline{a}\right] + (1 - \nu)\left[\left(1 - \overline{\theta}\right)\overline{p} - \overline{\theta}\overline{a}\right] \\ &= K - \nu\left[(1 - \underline{\theta}) h(\underline{u}_N) + \underline{\theta} h(\underline{u}_A)\right] - (1 - \nu)\left[\left(1 - \overline{\theta}\right) h(\overline{u}_N) + \overline{\theta} h(\overline{u}_A)\right], \end{aligned}$$

where

$$K \overset{\text{def}}{=} w - \left[\nu\underline{\theta} + (1 - \nu)\overline{\theta}\right] d.$$

We capture the fact that the market is characterized by perfect competition by assuming that the contracts in the menu maximize A's expected utility, subject to the constraint that P, in expectation, does not make a loss. In addition, to take into account that there is asymmetric information about A's type, we require the two incentive compatibility constraints to hold. That is, the contracts in the menu solve the following problem:

$$\max_{\underline{u}_N, \underline{u}_A, \overline{u}_N, \overline{u}_A} \nu\left[(1 - \underline{\theta})\underline{u}_N + \underline{\theta}\underline{u}_A\right] + (1 - \nu)\left[\left(1 - \overline{\theta}\right)\overline{u}_N + \overline{\theta}\overline{u}_A\right]$$

subject to

$$K - \nu\left[(1 - \underline{\theta}) h(\underline{u}_N) + \underline{\theta} h(\underline{u}_A)\right] - (1 - \nu)\left[\left(1 - \overline{\theta}\right) h(\overline{u}_N) + \overline{\theta} h(\overline{u}_A)\right] \geq 0, \qquad \text{(Profit)}$$

$$\left(1-\overline{\theta}\right)\overline{u}_N+\overline{\theta}\overline{u}_A \geq \left(1-\overline{\theta}\right)\underline{u}_N+\overline{\theta}\underline{u}_A, \qquad \text{(IC-high)}$$

$$\left(1-\underline{\theta}\right)\underline{u}_N+\underline{\theta}\underline{u}_A \geq \left(1-\underline{\theta}\right)\overline{u}_N+\underline{\theta}\overline{u}_A. \qquad \text{(IC-low)}$$

Let the second-best utility levels (i.e., the utility levels at the solution to the above maximization problem) be denoted by a superscript "SB": $\underline{u}_N^{SB}$, $\underline{u}_A^{SB}$, $\overline{u}_N^{SB}$, and $\overline{u}_A^{SB}$.

(a) Solve as much as you need of the above problem to show how $\underline{u}_N^{SB}$ relates to $\underline{u}_A^{SB}$, and how $\overline{u}_N^{SB}$ relates to $\overline{u}_A^{SB}$. You do not need to show that the second-order condition is satisfied (and if you nevertheless do that, you will not get any credit).

(b) In the course, we studied a model of a *monopoly* insurance market with adverse selection. Discuss how the economic logic of that model and the one described in this question differs (if you think there is anything in the economic logic that differs). In particular, in each one of the two models, what is the tradeoff that the principal faces, and how can that information help us understand how the optimal contracts look like in the two models?

End of Exam